Tidal Forces and their Effects in the Solar System

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Introduction

For most residents of Earth, tides are synonymous with the daily rise and fall of sea levels, and there is a general level of awareness that they are "caused by the pull of the Moon's gravity", with some puzzling questions such as how the *pull* of the Moon can cause a high tide on the *far* side of the Earth. Tidal forces are, in fact, more complex than a simple one-directional pull, and they are responsible for many effects in our solar system, from changing sea levels to geological activity on, and the ultimate fate of, distant moons.

What Are Tidal Forces?

Newton's law of gravity defines the attractive force between two bodies as

$$F = G\left(\frac{m_1 m_2}{r^2}\right),$$

treating the bodies as zero-dimensional points, but real objects such as planets and satellites have significant non-zero diameters. The inverse-square force of gravity is significantly different between points on the near sides of two bodies and between points on their far sides.

For orbital purposes, objects are considered gravitational point sources. For example, the Moon's orbit around Earth is described by Kepler's 3rd law which, in Newton's format, is

$$P^2 = \left[\frac{4\pi^2}{G(m_{earth} + m_{moon})}\right]a^3,$$

where P and a are period and semi-major axis. In other words, the Moon's position relative to Earth is determined by applying the law of gravity to the centre of the Moon and the centre of the Earth, ignoring their radii.

If we now consider the Moon to have radius *"w"*, the point on the Moon closest to Earth feels a gravitational attraction of

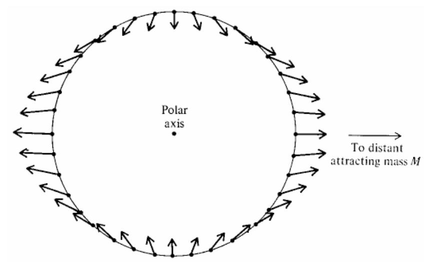
$$F = G\left(\frac{m_{earth} m_{moon}}{(r - w)^2}\right)$$

while the point farthest from Earth feels a gravitational attraction of

$$F = G\left(\frac{m_{earth} m_{moon}}{(r + w)^2}\right).$$

The near point is being pulled *more strongly* toward Earth, while the far point is being pulled *less strongly*, than the centre. Relative to the centre, which determines the Moon's orbital position, the near point is pulled *toward* Earth, while the far point is pushed *away* from Earth. The difference between these forces is the tidal force.

Barger and Olson [1973] show that if you perform this calculation for every point on the Moon, you get the vectors of force, relative to the centre, shown here.



The Moon exerts similar forces on Earth. Each body is stretched along an axis pointing toward the other, with the near side being pulled closer and the far side being pushed away, resulting in a "bulge" on both the near and far sides.

The tidal force pulling a satellite apart in the long direction is approximated by:

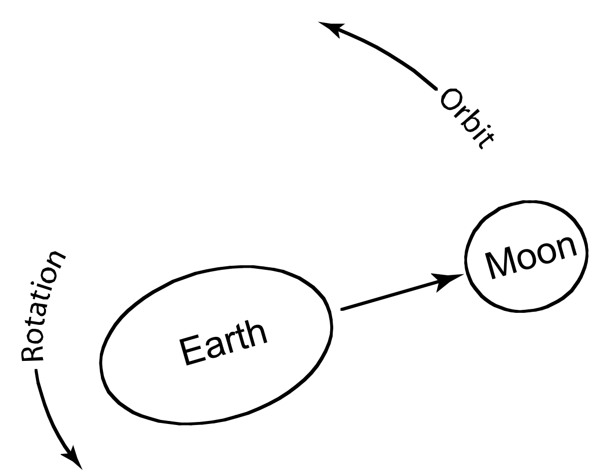
$$F = \frac{2GMmr}{R^3}$$

where M and m are the masses of the primary and satellite, R is the centre-centre distance, and r is the radius of the satellite [Freedman 2002, p 215]. This inverse *cube* function decreases rapidly with distance. Using known masses, we can estimate tidal forces for familiar bodies as:

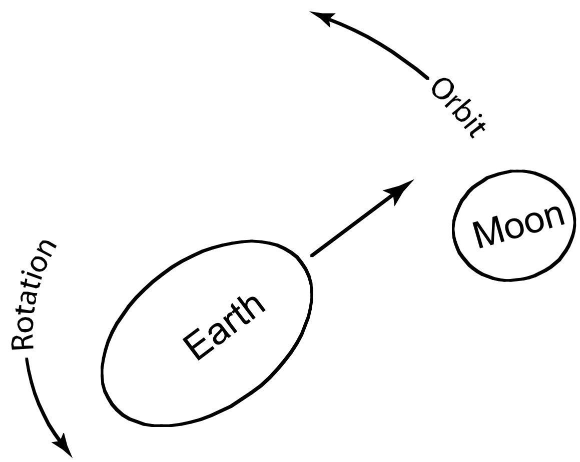
Moon feels from Earth	1.82×10^{18} N
Earth feels from Moon	6.69×10^{18} N
Earth feels from Sun	3.02×10^{18} N

Earth feels 3.5 times more tidal force than the Moon from their relationship, because the Earth is bigger, and thus has a bigger differential. The Sun's effect on Earth is only half the Moon's because, while more massive, it is much farther away.

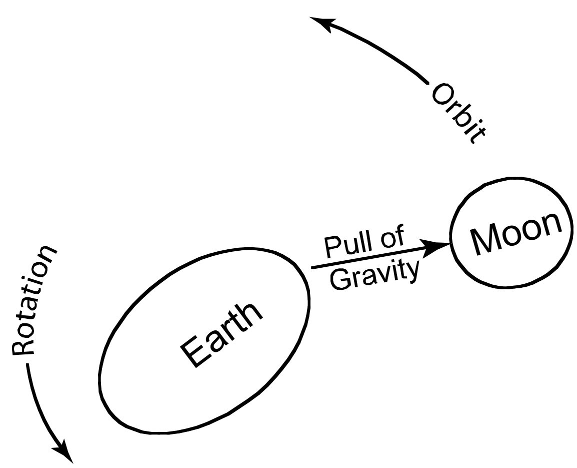
Now let's consider that the Earth is also *rotating*. If Earth were rotating at the same rate as the Moon's orbital period, the bulge would point straight at the Moon:



However, the Earth is rotating considerably faster than the Moon's orbital period, and the bulge, which can't move as quickly, is pushed ahead, and points ahead of the Moon's position:



The Moon's gravity "pulls back" on the advanced bulge:



This backward torque gradually slows the rotation of the earth. To conserve the angular momentum of the system, the radius of the Moon's orbit gradually increases. If the Earth were rotating more *slowly* than the Moon's orbit, the bulge would lag *behind* the connecting line, and the Moon would be gradually speeding up the Earth's rotation, and moving closer to conserve angular momentum.

All of these effects also happen to the Moon in response to the Earth's gravitational pull[1].

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[1] In fact, neither Earth nor Moon orbits the centre of the other. Both orbit the *barycentre* – the common centre of gravity between the two. The simple assumption of one body orbiting the other is sufficient for illustration.

Effects of Tidal Forces

The effects of tidal forces are seen on Earth as changing sea levels, and as a variety of phenomena throughout the solar system.

Tides on Earth

The tidal bulge, when it is over an ocean, presents itself as a rise in sea level. 90° later in revolution, when the bulge is at right angle to the ocean, a reduced sea level is experienced.

The Moon is not alone in pulling on the Earth. The distant Sun is so massive that it also exerts a significant tidal force, about half the strength of the Moon's. At Syzygy, when the Earth, Moon, and Sun are in a straight line (Full Moon and New Moon), the Sun's effect is added to the Moon's, creating the highest tides, called Spring Tides[2]. At Quadrature, when the Sun and Moon form a 90° angle with Earth, the Sun's effect is minimized, and the lowest tides, Neap Tides[3], occur.

Resonances

Tidal forces cause the rotations and orbits of bodies to be synchronized in integer ratios. The most evident case is *Synchronous Rotation*, where a satellite rotates at the same rate as its orbital period, keeping the same face toward its parent. All large satellites in our solar system, including our Moon, do this. Above, we discussed the slowing of Earth's orbit by the Moon pulling on the advanced tidal bulge. In the distant past, when the Moon was rotating at some other speed, Earth would have had the same effect on the Moon, slowing or speeding its rotation until it reached a rotational speed where the bulge precisely faced the Earth.

Veverka [1987] points out that "one would expect that only remote satellites might not spin synchronously" and confirms that, for example, all of Jupiter's satellites from Callisto and inward are tidally locked, while those outside Callisto are not.

Higher-Order Resonances also occur in the solar system. For example, the orbits of Io, Europa, and Ganymede are in a 1:2:4 resonance. Relationships such as these result from interaction of the bodies' gravitational forces on one another, selecting timings that correspond to wave functions with integer solutions.

It was long assumed that Mercury was tidally locked (1:1) with the Sun [Chaisson, 2005] but, in 1965, Doppler radar revealed that Mercury is, in fact, in a 3:2 spin-orbit

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[2] From Middle English "springan", *springing up*.

[3] From old English "nepflod" and Dutch "nipen", to be "nipped off".

resonance. 40 years later Correia and Laskar [2004] showed the relationship could be explained by using the equations for an oscillating damped pendulum.

Tidal Heating

Tidal forces can also internally heat a satellite. Satellites rotate at a constant speed but, in an eccentric orbit, the orbital speed varies with the distance from the primary (Kepler's 2nd law). Thus the tidal bulge of a tidally locked satellite cannot always point precisely at its primary. At some points in the orbit, the bulge will be pointed ahead of or behind where it should be, and the primary will be exerting a small pull on the bulge. The friction of the bulge being pulled back and forth through the solid body of the satellite heats the interior of the body.

Io is an extreme example of tidal heating. Peal et al [1979] pointed out that, while Io's orbit is nearly circular, it frequently encounters Europa and Ganymede, causing temporary changes in the tidal forces it feels, moving the tidal bulge around the planet. In the Jupiter's intense gravity the friction is significant, and they predicted Io should be hot enough for Volcanism. One week later, Voyager-1 photographed active volcanoes on Io, and we now know it to be the most volcanically active object in the solar system, with internal temperatures of 2000K [Chaisson, 2005].

Tidal heating is also probably responsible for keeping the interior of Europa warm enough for the liquid water that is suspected to exist below the ice surface, and tidal resonance with Saturn's Dione is thought to power volcanism on Enceladus.

The Roche Limit

In 1848, Astronomer Edouard Roche noted that, if a satellite was held together mainly by its own gravitational attraction, there would be a minimum distance from the primary inside which the tidal forces of the primary would exceed the satellite's binding forces and would tear it apart [Hoskin, 1996].

The Roche Limit for two bodies is approximated by a function of their densities:

$$L = 2.456R\left(\frac{r_{planet}}{r_{satellite}}\right)^{\frac{1}{3}}$$

where R is the planet's radius, and the *r* values are the densities. For typical satellites, a common approximation is that the Roche Limit is 2.5 times the radius of the primary planet.

Darling [2004] gives Roche Limit values for some planets and normal satellites as:

Earth	18,470 km
Jupiter	175,000
Saturn	147,000
Uranus	62,000
Neptune	59,000

The Roche Limit applies only to fluid bodies held together entirely by gravitation. Small satellites and moons can survive inside their primary's Roche Limit because their electrochemical bonds are more significant than their gravitational bonds. For small rocky satellites, the Roche Limit is approximated as 1.44 primary radii.

Rings and Satellites

All large satellites in the solar system orbit outside their planet's Roche Limit. Small rocky satellites (usually under 100 km in diameter) can exist inside the Roche Limit, and there are examples of this with satellites of Jupiter, Uranus, and Neptune.

All the Jovian planets are now known to have rings, all *inside* their primary's Roche Limit, when the Roche calculations for a relatively diffuse body are used.

Maxwell [1859] proved that Saturn's rings must be composed of small particles, and Roche suggested they were a former satellite that broke up because it was inside the Roche Limit. However, tidal forces would have prevented a satellite from forming in the first place, so a mechanism to form a satellite elsewhere and then move it inside the limit was needed. It is possible that a separate object was captured by Saturn's gravity and pulled inside the Roche Limit, where it was destroyed (as happened with comet Shoemaker-Levy 9 at Jupiter), or that a moon, originally further out, spiralled into the planet through tidal interactions (as is happening with Phobos and Triton).

An alternate theory is that the rings consist of original material from the solar nebula, prevented by tidal forces from ever becoming a satellite. In this case, a mechanism is needed to explain the sharp boundaries of the ring systems, and why they have not become diffuse with time. Shepherding effects of small nearby satellites are the proposed solution to the sharp boundaries, and replenishment of the rings with dust stripped off satellites could keep them from fading with time. Galileo photographs from 1996-97 show dust being pulled off Amalthea and Thebe and into the rings of Jupiter, supporting this theory.

Catastrophic Events

Finally, occasional extraordinary events serve to demonstrate the power of tidal forces.

Neptune's moon Triton is due for such an event. Like all retrograde satellites, its orbit is decaying, and it will fall below Neptune's Roche limit and be destroyed in 100 Million to 1 Billion years.

More recently, when comet Shoemaker-Levy 9 was discovered in 1993, it had already broken up into more than twenty pieces when it passed within 21,000 km of Jupiter. (Jupiter's Roche limit is 175,000 km, so the tidal stresses at 21,000 would have been enormous.) The pieces spiralled in to impact the planet in July 2004. This event showed that theories of stray objects being destroyed and contributing to planetary evolution are credible.

Conclusion

Tidal forces result from the difference in the force of gravity at different points on a moving body. They are common in the solar system, causing changing sea levels and also being the influence and power source for a great variety of the dynamic processes that helped form and continue to shape it.

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